TM-395
0350

SLITS IN THE 8-GeV TRANSPORT

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September 28, 1972

I. It has been suggested that horizontal and vertical slits should be placed in the 8-GeV transport in order to reduce the transverse beam emittance. Presently, the main ring accepts only a fraction of the injected beam. There must be many causes for this beam loss but it will be very useful to find out the effect of the transverse emittance of the injected beam on the subsequent beam loss in the main ring. The relevant quantity here is not the real emittance of the beam in phase space but an "effective" emittance. If only one slit is used in the horizontal (or vertical) direction, the beam emittance is reduced but the maximum excursion of the beam in the main ring remains unchanged. Because of its tune spread, the beam will eventually occupy the same phase space area.* One needs at least a pair of slits to achieve a reduction in the effective emittance. Ideally speaking, when the effective emittance is reduced to a fraction of the original

*It is assumed here that the mismatching at injection is small so that the real emittance is the same as the effective emittance when there is no slit.

value, one would like to maintain the intensity of the beam at the same fraction of its original value. Unless particles are concentrated near the origin of the phase space, this is not possible to achieve with simple slits. For a beam uniformly distributed in phase space, the intensity is proportional to the real emittance so that a figure of merit (FM) of the slit performance can be defined as

$$FM = \frac{\text{real emittance}}{\text{effective emittance}} \text{ at injection.}$$

II. Consider a beam with the emittance πE. At the location of the first slit, betatron oscillation parameters are assumed to be α_1 and β_1. The beam size is

$$\pm x_{1m} = \pm\sqrt{\beta_1 E}.$$

The beam is limited by the first slit such that

$$-h_1 \leq x \leq h_1,$$

$$h_1 = \varepsilon_1 \, x_{1m} = \varepsilon_1 \, \sqrt{\beta_1 E}.$$

At the location of the second slit, the slit size is $2h_2$. When ε_1 is small, the beam size at the second slit location is approximately

$$\pm x_{2m} = \pm |m_{12}| \, \sqrt{E/\beta_1} = \pm \left(|m_{12}| \middle/ \beta_1\right) \, x_{1m}$$

where m_{12} is an element of the transfer matrix M from the first slit to the second slit,

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} .$$

Also,

$$h_2 = \varepsilon_2 \, x_{2m}$$

defines the parameter ε_2. After passing two slits, the real emittance of the beam relative to the initial emittance is, for small ε_1 and ε_2,

$$\frac{\text{real emittance}}{\pi E} = (4/\pi)\ \varepsilon_1 \varepsilon_2 .$$

On the other hand, the effective emittance of the same beam relative to the initial emittance is

$$\frac{\text{effective emittance}}{\pi E} = (k\varepsilon_1 + \varepsilon_2)^2 + \varepsilon_1^2$$

where*

$$k = \left| (m_{11}\beta_1/m_{12}) - \alpha_1 \right| .$$

This expression is valid if

$$\varepsilon_1 << 1/k$$

and

$$\varepsilon_2 < 1 - k\varepsilon_1 .$$

The derivation is lengthy but straightforward.

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*The phase advance $\Delta\psi$ of the betatron oscillation from slit 1 to slit 2 is related to k by

$$\left| \tan\ (\Delta\psi) \right| = 1/k .$$

III. The beam size at slit locations should not be too small. Otherwise slit sizes (h_1 and h_2) become impractical for small values of ε_1 and ε_2. For a given real emittance, that is, for $\varepsilon_1\varepsilon_2$ = const., FM takes the maximum value

$$(FM)_{max} = \frac{4}{\pi} \frac{\sqrt{1+k^2}}{1 + \left(k + \sqrt{1+k^2}\right)^2}$$

when

$$\varepsilon_2 = \varepsilon_1 \sqrt{1+k^2} \ .$$

It is obvious that the value of the parameter $k(\geq 0)$ should be as small as possible. The best one can do is

$$FM = 2/\pi = 0.637 \text{ for } k = 0.$$

Note that one is using an approximation that is valid only if

$$k\varepsilon_1 << 1.$$

Values of FM relative to the best value are shown in Fig. 1 as a function of k.

Suitable locations for slits in the 8-GeV transport have not been searched exhaustively in accordance with the criteria discussed above. However, the following examples will show the degree of success one can hope for.

(A) <u>Horizontal</u>

Beam emittance assumed: $\pi E = 2\pi$ mm-mrad

Slit 1: downstream end of MQ-43

x_{1m} = 14.2 mm

Slit 2: upstream end of MQ-50

$x_{2m} = 11.4$ mm

$k = 0.0167$

With $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.5$,

$h_1 = 7.1$ mm, $h_2 = 5.7$ mm.

Real emittance = 0.318 (πE) = 0.637 π mm-mrad

Effective emittance = 0.508 (πE) = 1.017 π mm-mrad

FM = (0.637/1.017) = 0.626

The performance is almost perfect.

(B) Vertical

Beam emittance assumed: $\pi E = 2\pi$ mm-mrad

Slit 1: downstream end of MQ-15

$y_{1m} = 21.8$ mm

Slit 2: downstream end of MQ-46

$y_{2m} = 7.67$ mm

$k = 0.2218$

With $\varepsilon_1 = 0.3$ and $\varepsilon_2 = 0.4$,

$h_1 = 6.5$ mm, $h_2 = 3.1$ mm

Real emittance = 0.153 (πE) = 0.306π mm-mrad

Effective emittance = 0.308 (πE) = 0.615π mm-mrad

FM = (0.306/0.615) = 0.497.

The choice of ε_2 is not optimum here* and the performance is inferior to what one gets in the horizontal direction.

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*The optimum choice is: $\varepsilon_2 = 0.307$, $h_2 = 2.36$ mm and FM = 0.511.

Throughout the discussion in this report, the increase of the beam size due to the momentum dispersion has been ignored. More serious defects of the analysis will undoubtedly arise from misalignments which unfortunately are not at all insignificant in the 8-GeV transport.

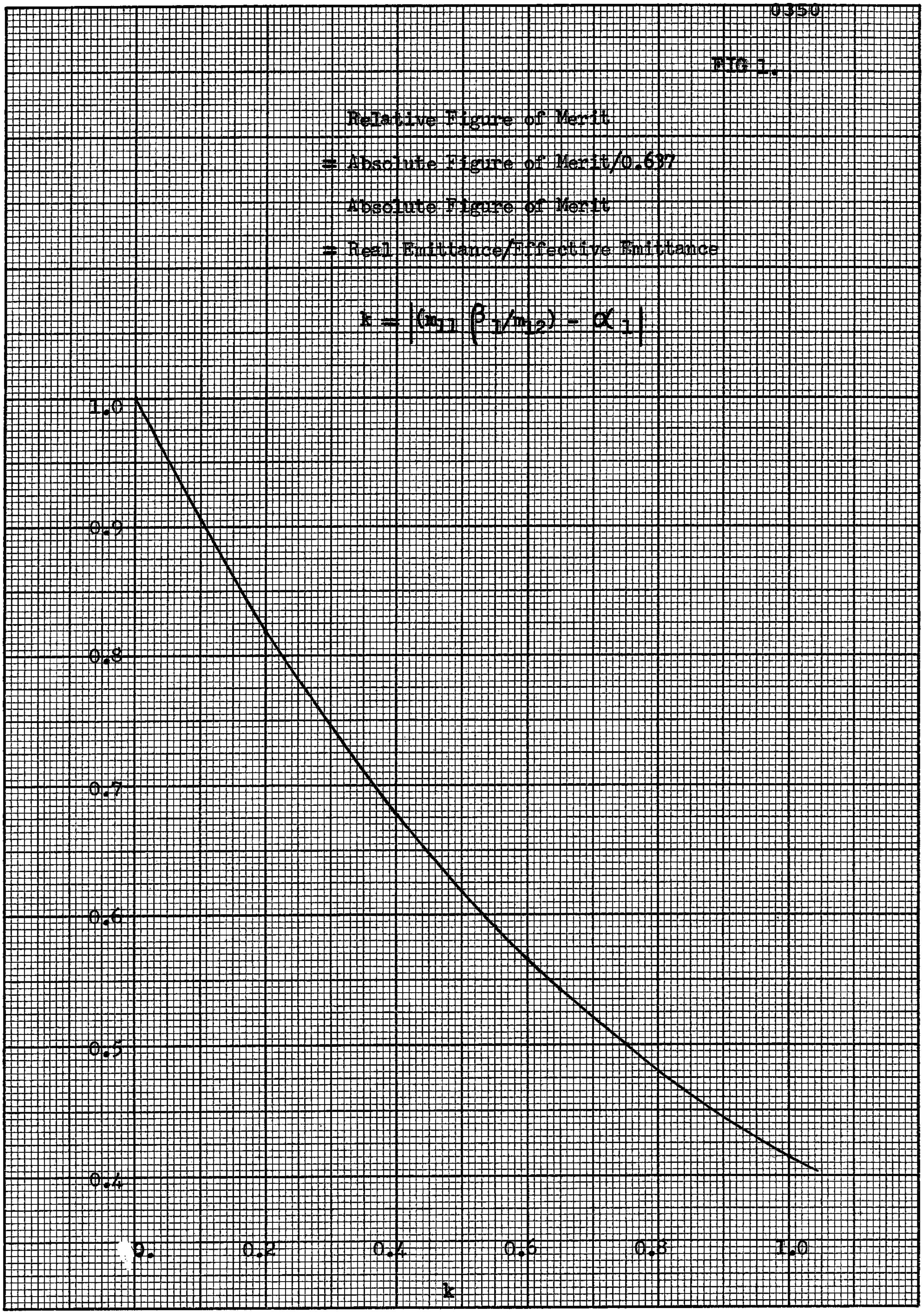
FIG 1.
Relative Figure of Merit
= Absolute Figure of Merit/0.637
Absolute Figure of Merit
= Real Emittance/Effective Emittance
$k = \left|(m_{11}\left(\beta_1/m_{12}\right) - \alpha_1\right|$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.
0.2
0.4
0.6
0.8
1.0
k

ADDENDUM

Initial beam emittance = πE

Maximum beam size at the first slit = $\pm\sqrt{\beta_1}\ \sqrt{E}$

at the second slit = $\pm\left(|m_{12}|/\sqrt{\beta_1}\right)\sqrt{E}$

UE ≡ upstream end; DE ≡ downstream end

(A) Horizontal

1. Slit 1: midpoint between MQ-15 and MQ-40, $\sqrt{\beta_1}$ = 11.11 $m^{1/2}$

	Position	$\lvert m_{12}\rvert/\sqrt{\beta_1}$	$\sqrt{1+k^2}$	Relative FM
Slit 2:	UE MH-50	7.111$m^{1/2}$	1.0119	0.857
	DE MH-50	7.469	1.0167	0.833
	UE MQ-50	7.889	1.0226	0.809
	DE MQ-50	7.247	1.0274	0.792
	UE MQ-51	6.767	1.0293	0.785
	DE MQ-51	5.974	1.0379	0.760
	UE MV-60	5.915	1.0505	0.729
	DE MV-60	5.854	1.0648	0.699

2. Slit 1: UE MQ-40, $\sqrt{\beta_1}$ = 13.25 $m^{1/2}$

Slit 2:	UE MH-50	7.195	1.0001	0.990
	DE MH-50	7.592	1.0002	0.983
	UE MQ-50	8.058	1.0011	0.954
	DE MQ-50	7.428	1.0023	0.934
	UE MQ-51	6.946	1.0029	0.927
	DE MQ-51	6.164	1.0058	0.898
	UE MV-60	6.146	1.0110	0.862
	DE MV-60	6.125	1.0177	0.829

3. Slit 1: DE MQ-43, $\sqrt{\beta_1} = 10.01\ m^{1/2}$

	Position	$\lvert m_{12}\rvert/\sqrt{\beta_1}$	$\sqrt{1+k^2}$	Relative FM
Slit 2:	UE MH-50	$7.189 m^{1/2}$	1.0008	0.960
	DE MH-50	7.593	1.0001	0.988
	UE MQ-50	8.066	1.0001	0.983
	DE MQ-50	7.440	1.0007	0.963
	UE MQ-51	6.958	1.0010	0.955
	DE MQ-51	6.181	1.0030	0.926
	UE MV-60	6.171	1.0069	0.889
	DE MV-60	6.157	1.0124	0.854
	UE MV-61	6.062	1.0724	0.685

(B) Vertical

1. Slit 1: UE MQ-02, $\sqrt{\beta_1} = 7.978\ m^{1/2}$

Slit 2:	DE MQ-12	$7.678 m^{1/2}$	1.0617	0.705
	UE MQ-13	9.647	1.0667	0.695
	DE MQ-13	10.80	1.0710	0.688

2. Slit 1: DE MQ-02, $\sqrt{\beta_1} = 8.371\ m^{1/2}$

Slit 2:	UE MQ-12	$6.282 m^{1/2}$	1.0460	0.739
	DE MQ-12	7.729	1.0547	0.719
	UE MQ-13	9.713	1.0594	0.710
	DE MQ-13	10.87	1.0634	0.702
	DE MQ-15	14.40	1.0705	0.689

3. Slit 1: UE MQ-03, $\sqrt{\beta_1} = 7.411\ m^{1/2}$

Slit 2:	DE MQ-12	$7.809 m^{1/2}$	1.0439	0.744
	UE MQ-13	9.819	1.0480	0.734
	DE MQ-13	11.00	1.0516	0.726
	DE MQ-15	14.57	1.0579	0.713

4. Slit 1: UE MQ-13, $\sqrt{\beta_1} = 10.29\ m^{1/2}$

	Position	$\lvert m_{12}\rvert/\sqrt{\beta_1}$	$\sqrt{1+k^2}$	Relative FM
Slit 2:	UE MQ-46	$5.085m^{1/2}$	1.0278	0.790
	DE MQ-46	5.459	1.0183	0.826

5. Slit 1: DE MQ-15, $\sqrt{\beta_1} = 15.42\ m^{1/2}$

Slit 2:	UE MQ-46	5.048	1.0352	0.767
	DE MQ-46	5.427	1.0243	0.803

6. Slit 1: Midpoint between MQ-15 and MQ-40, $\sqrt{\beta_1} = 12.82\ m^{1/2}$

Slit 2:	DE MQ-46	5.239	1.0611	0.706